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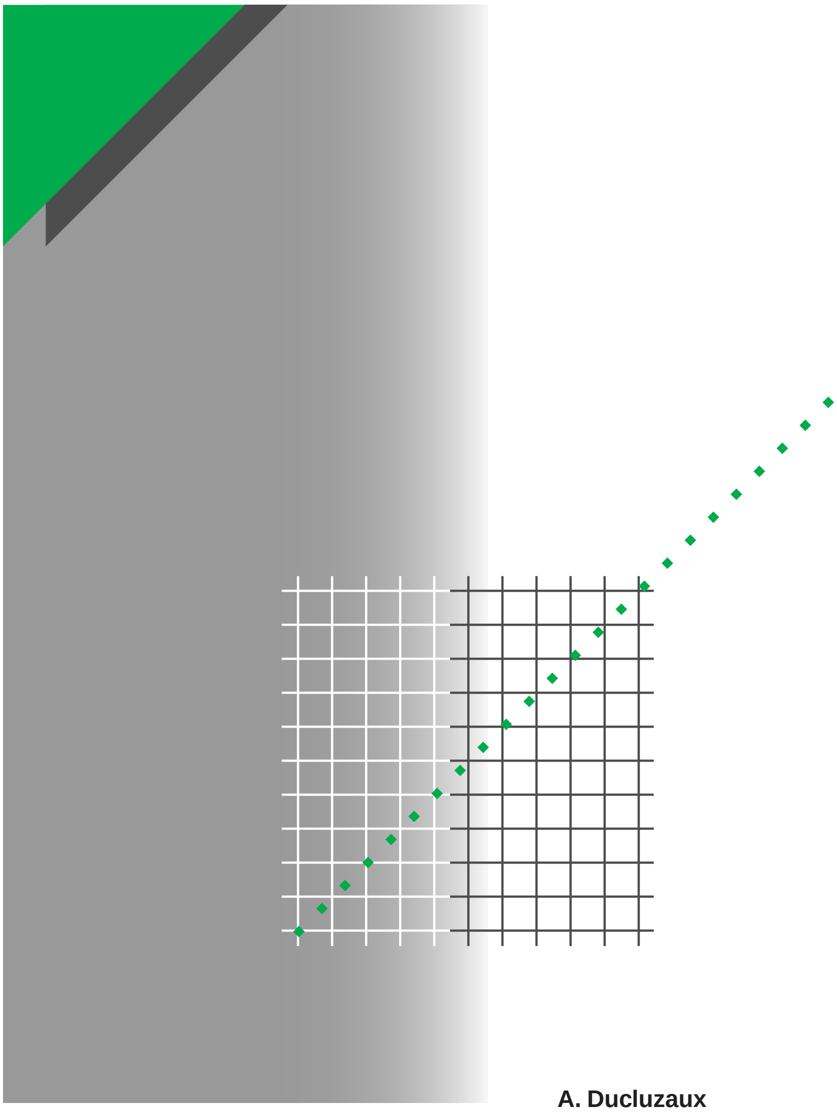
## Extra losses caused in high current conductors by skin and proximity effects



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**Foreword**

# no. 83

# Extra losses caused in high current conductors by skin and proximity effects

**André DUCLUZAUX**

**Engineering diploma from ESME in 1950, bachelor of Science in 1952, he began working at Merlin Gerin in 1952. He first participated in the study of LV factory assembled switchboards then perfection of switchgear in the high power testing station.**
**In 1960, as head of the research office for LV high current circuit-breakers, he developed DA circuit-breakers, and was then in charge of LV studies. Since 1969, he has been in charge of projects in the general research department and has provided technical training and information sessions.**
**It was whilst carrying out these functions that he made more detailed studies of the skin and proximity effects for the development of apparatus and high current busbars and condensed the most essential information on this subject into a practical document.**

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# Extra losses caused in high current conductors by skin and proximity effects

A century ago, Lord Kelvin showed that any rapid change of current intensity in a conductor modifies the current density in the various pans of this conductor. The author recalls the consequences of the skin and proximity effects in the case of electrical conductors designed for high currents.

More attention should be made to these phenomena in the case of the designing of certain busbars since it is clear that this particularity, often neglected, leads to an overdimensioning of the conductors, higher energy losses and poor overall operational efficiency.

## Contents

# 1 Introduction

Electricians have known for a little over a century, since 1873, that alternating currents have a preference for moving at the periphery of solid conductors.

In itself, this characteristic would not be a nuisance if it did not lead to extra losses of energy. In a solid conductor, the losses and the heating take place as if the effective A.C. resistance were higher than the actual D.C. resistance.

The increase in resistance which can be of the order of 10 to 20% for conductors designed for 2000 A, grows much faster than the increase in cross section for carrying higher currents.

This results in two disadvantages:

- Waste of electric energy through supplementary losses; more recently industrialists have recognised that these latter represent a luxury which is not limited only to financial considerations.
- Waste of raw materials, copper or aluminium, because of the larger amounts of metals used, and badly used, as electrical conductor.

Energy losses in the relatively short conductors of distribution equipment are generally only taken into account as far as their physical effects are concerned: heating and evacuation of calories. However the economic aspects of the energetic efficiency of a conductor are far from negligible at low voltages; it is easy to show that an assembly of conductors having a cross section of 1000 mm$^2$, carrying 2000 A, loses, in a year of continuous use, an amount of energy whose cost is about the same as that of the copper they contain.

An extra loss of 10% due to the skin effect thus represents the price of the copper for the whole lifetime of the installation (20 years with a working factor of 0.5).

Kelvin's law states in fact that the economic cross section of the copper (or aluminium) which has to be used for conductors is that for which the cost of the annual Joule losses is equal to the annual amortisation charges for the copper, plus the other constructional elements proportional to the weight of the copper.

It is of course the role of Merlin Gerin, a manufacturer of switchgear and distribution equipment, to have complete command of the technical problems involved in the design of that equipment. However the job of a manufacturer is not limited merely to supplying products; he owes it to his clients to make available his technical expertise to enable them to install and use the equipment in the best possible way.

The aim of this article will thus be to give the principle and to evaluate the consequences of the skin and proximity effects, and to summarize practical data which can be useful to the responsible for installing busbars carrying high current. These effects start becoming significant for conductors of 1600 to 2000 A, but become very important above 4000 to 5000 A.

# 2 The skin effects

## 2.1 Generalities

The most striking aspect of the skin effect is the increase in the current density at the periphery of A.C. solid conductors, but this does not explain at all the increase in the effective resistance.

One explanation often put forward is that the inductance of the central filament of current in a conductor is higher than those of the peripheral filaments, this inductance being linked to the variation in the enveloped flux, which is itself a maximum for the central current filament.

In order to equalise the drops in inductive voltages between the various filaments, a higher current flows in the peripheral filaments. These different currents are thus more or less out of phase and their arithmetic sum is higher than the total measured current ; as a result there are extra losses due to the Joule effect. This is equivalent to saying that the effective resistance is increased.

In order to get a better quantitative idea of this skin effect, and of its implications, it is necessary to use the mathematical arguments developed by Lord Kelvin in 1889 [1] (1) based on the propagation equations established by Maxwell a few years earlier.

These demonstrations, which appear in particular in references [3], [9], [13], [14], [20], [24], are outside the scope of this study; here an attempt will simply be made to describe the skin effect and its consequences by a qualitative reasoning based on the induced parasitic currents, so called “eddy-currents”, with the help of simple vector diagrams.

Consider a solid rectangular conductor (see **fig. 1** ) which is made up along part of its length of 3 elements: 1 and 3 at the periphery and 2 in the centre.

For a direct current, the total current I passing through the conductor is the sum of the 3 currents which are identical in each conductor:

$I = I_1 + I_2 + I_3$

For an alternating current, induced currents are produced in addition to the three previous currents.

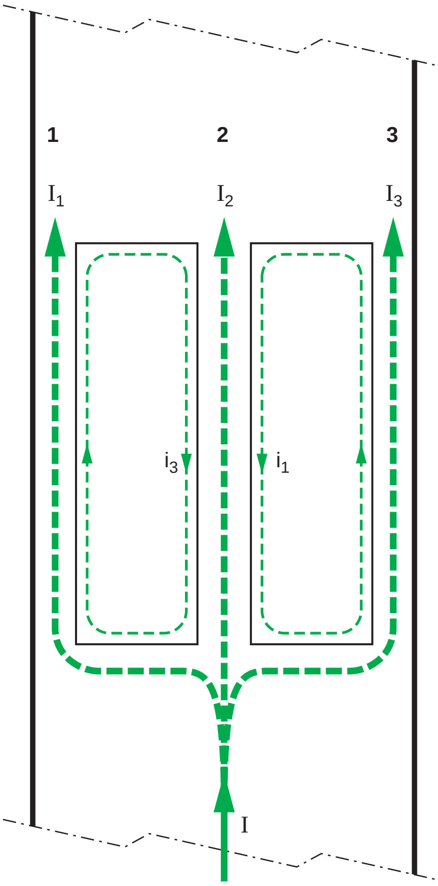


*Fig. 1*

Element 3, carrying $I_3$, induces in the rectangle made up of 1 and 2 an e.m.f. $e_3$ which produces a current $i_3$, with a phase shift of $\alpha$ (close to $\pi/2$). The resulting current in element 1 is:

$\vec{I'_1} = \vec{I_1} + \vec{i_3}$

---

(1) Numbers between [] concern the bibliography.

It is observed from the vector diagram (see **fig. 2** ) that $I'_1$ has a higher value than $I_1$ and is out of phase, in front of $I_1$, a vector considered to be in phase with the voltage U existing between the extremities of the 3 elements which are at the same potential.

In the same way, in element 3:

$$\vec{I'_3} = \vec{I_3} + \vec{i_1}$$

However, the resulting current $I'_2$ in the central element 2 will be decreased because of the two induced currents $i_1$ and $i_3$:

$$\vec{I'_2} = \vec{I_2} - \vec{i_3} - \vec{i_1}$$

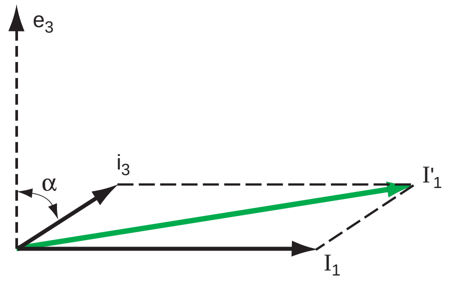


*Fig. 2*

The diagram of **figure 3** shows that the resulting current $I'_2$ has in fact a lower amplitude than $I_2$ and is lagging.

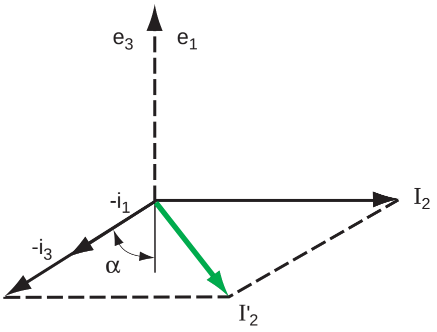


*Fig. 3*

In **figure 4** , the total current I in the whole conductor appears as the vectorial sum of the partial currents in the 3 elements considered:

$$\vec{I} = \vec{I'_1} + \vec{I'_2} + \vec{I'_3}$$

If this reasoning is transposed to all of the current elements of the solid conductor, and not just to three of them, the general aspect of the phenomena remains the same; there is a gradual change in the current intensity and in the phase shift of the current elements from the periphery to the centre. The causes and the consequences of the skin effect relative to the different electrical and physical parameters have thus been demonstrated:

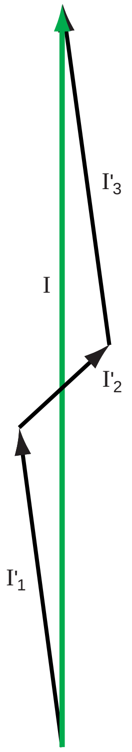


*Fig. 4*

- Current density

The density at the periphery is higher than the average current density ( $I'_1 > I_1$),
the density of the current in the core is less than this average density ( $I'_2 < I_2$).

- Current intensity

The sum of the amplitudes of the currents in the different elements is greater than the total current (see fig. 4).

- Losses

The real losses due to the Joule effect are thus higher, this being expressed by considering that the effective A.C. resistance Ra is higher than the real D.C. resistance Rc, whence the extra losses.

In practice, the level of the skin effect, or the coefficient of resistance increase, or of the supplementary losses, is given by the ratio:

$$K = \frac{Ra}{Rc} \geqslant 1$$

- Phase shift

Compared to the voltage at the extremities of the conductor, the current at the periphery is leading, the core current lagging; the phase shift in the centre can reach and even exceed π/2, to such an extent that the residual current intensity has to be subtracted from the total current carried; it is in effect a totally parasitic current.

- Inductance

The effective inductance of an A.C. conductor is made up of two terms: the first $L_1$ is the inductance of the elements of the exterior circuits of the conductor, the second $L_2$ is the actual internal inductance resulting from the internal field. $L_2$ is a function of the current distribution at the interior; as this heterogeneous distribution consists in an increase in the density at the periphery, the term $L_2$ decreases. The skin effect therefore decreases the effective inductance of a conductor.

- Permeability

The foregoing argument is based on the phenomenon of induction. As a result, the permeability of the medium plays a conventional role; the skin effect is therefore much more pronounced in magnetic conductors of high permeability.

- Frequency

The increase in the skin effect with frequency results also from its origin related to an induction phenomenon, proportional to the flux variation.

- Resistivity

A higher resistivity of the conducting medium leads to a decrease in the induced currents, therefore to a less marked skin effect.

## 2.2 The imaginery shell

In trying to simplify the interpretation of the skin effect, Boucherot [2] proposed in 1905 the notion of an imaginary shell also called skin thickness or depth of penetration.

From the point of view of the Joule effect, what occurs in a solid conductor is the same as if all the current was transported in a peripheral layer, or shell, of thickness δ, the current density being uniform in this shell and zero at the centre:

$$\delta = \frac{1}{2\pi}\sqrt{\frac{10\,\rho}{\mu f}}$$

where:

δ: thickness of the shell in metres

ρ: resistivity in Ω/m

μ: permeability, 4π x $10^{-7}$ for a vacuum

f: frequency in Hz

In reality, the density decreases exponentially from the periphery to the centre of the conductor. At a depth of δ, the density is still

$\frac{1}{e} = 0.367$ as shown in **figure 5** .

The notion of an imaginary shell assumes that the average density in the shell is equal to $\frac{1}{\sqrt{2}}$ times the peripheral density.

From a practical point of view, the shell or the depth of penetration makes it possible to see very quickly whether the metal of a conductor is used correctly knowing the 3 values ρ, μ and f.

At 50 Hz copper has a skin of 8.5 mm, aluminium of 10.5 mm; this shows that it would be a waste to use a bar thickness, or a rod diameter, greater than for copper, or 20 mm for aluminium.

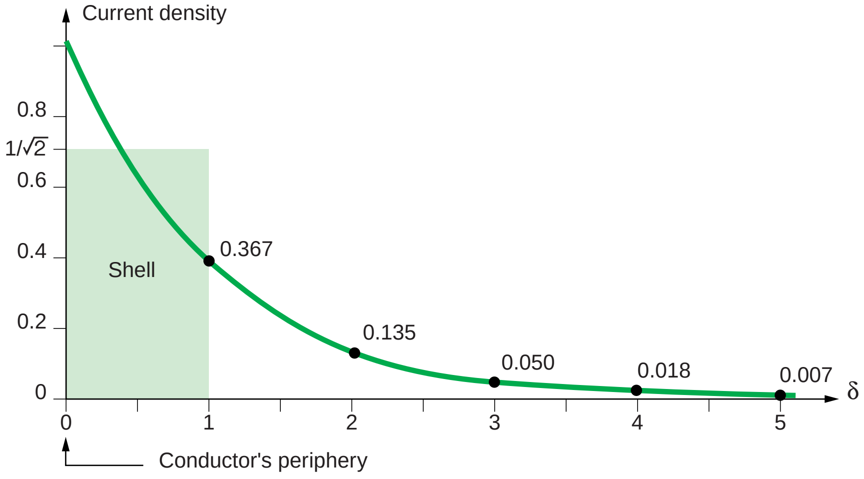


***Fig. 5*** *: decrease of current density inside a conductor.*

For steel, the skin is about in the order of a millimeter, when it is not saturated; this indicates clearly the uselessness of using steel conductors of thickness greater than 2 mm, except for mechanical reasons.

It should be noted that a progressive saturation causes the currents to penetrate more deeply into the steel conductors; this has enabled some original applications to be proposed, such as a variable resistance based on this phenomenon [7].

## 2.3 The skin effect in cylindrical conductors

For this particular shape, the calculations are less complex and the results more accurate. The only parameter which is generally considered is the ratio $K = \frac{Ra}{Rc}$ or the extra loss coefficient which indicates an inadequate conductor shape when its value diverges too much from unity. Several empirical formulae have been proposed; that of Levasseur [6] is particularly simple and leads to errors of less than 2%:

$$K = \sqrt[6]{\left(\frac{3}{4}\right)^6 + \left(\frac{S}{p\delta}\right)^6} + 0.25$$

Where S is the cross-sectional area of the conductor, p its perimeter, and $\delta$ the skin thickness.

From the tables and charts published, we give in **figure 6** results obtained from Dwight's original work [3].

The value of K as a function of the D.C. resistance Rc is obtained for each shape characterized by the ratio of the thickness to the diameter. These curves are thus valid for any resistance of the metal (non-magnetic). The reference frequency is 50 Hz, for any other frequency f, it is sufficient to replace the value of Rc by $Rc\,\frac{50}{f}$.

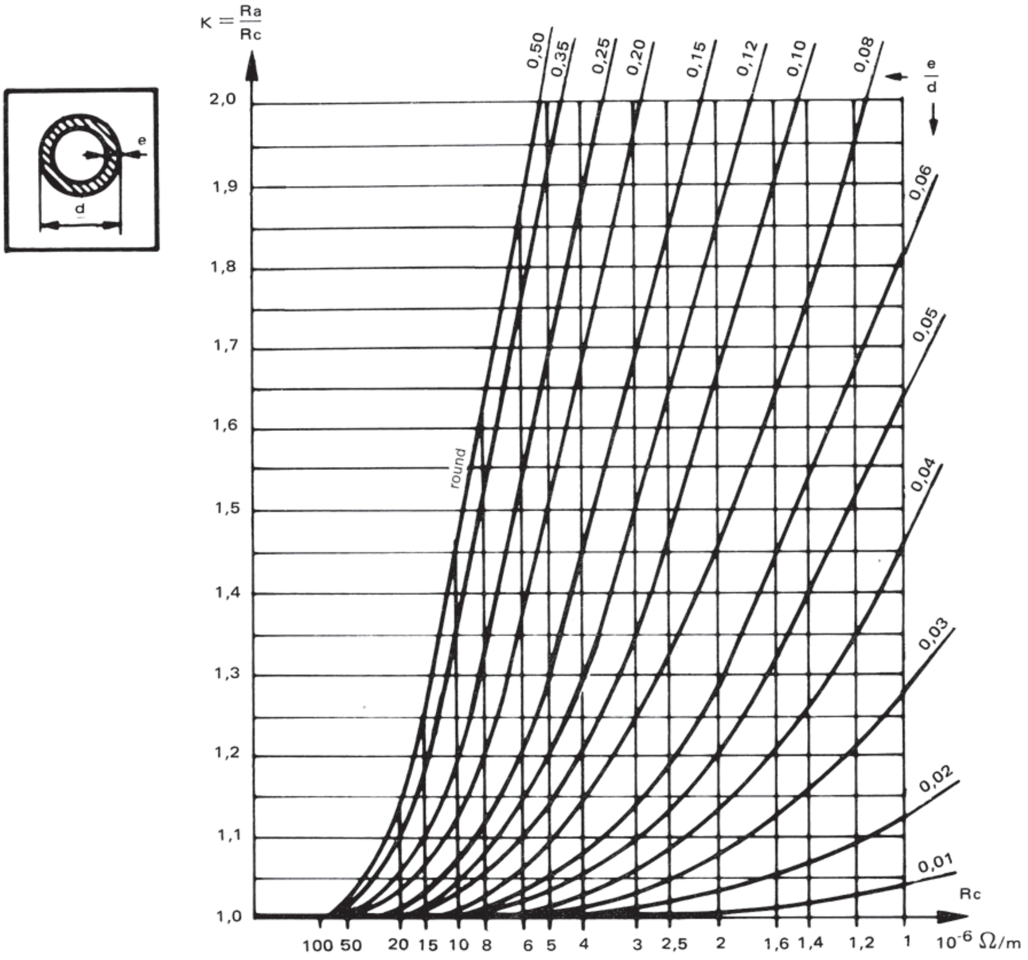


*Fig. 6 : extra loss coefficient for the skin effect in cylindrical conductors.*

The cylindrical conductors of high cross-sectional area usually found in practice are tubes and cables.

The splitting up of a cable into thin wires to increase flexibility causes no changes in the skin effect, as can be deduced by analogy with the splitting of steel plates into thin sheets to reduce magnetic circuits. In these plates, the eddy currents are transversal, whereas they are longitudinal in a cable. The splitting of a high cross-section cable into bunches of wires could be used to reduce its coefficient K, as long as the threads were regularly permuted, i.e. wound alternately at the periphery and at the center.

However sections of above 400 mm$^2$ for copper and 500 mm$^2$ for aluminium are rarely used; this corresponds to a 95% use of the metal.

## 2.4 Skin effects in a conductor of rectangular cross-section

In this case, the calculations are far more complex and remain inaccurate because of the assumptions made concerning the distribution of the magnetic field; authors who have attempted such calculations (see references) have often completed their work with delicate experiments.

In a qualitative way, **figure 7** (which is taken from the extensive study of Schwenkhagen [5] and confirmed by Renaud [13]), illustrates the importance of the phenomenon in a 100 x10 mm copper bar at 50 Hz.

The curves show, for each internal point situated on the axis, the density of the relative current with respect to the average density and the phase shift with respect to the voltage. However the determination of the coefficient K of extra losses by calculation or experiment remains unreliable in view of the varying values proposed for a 100 x 10 mm bar of copper: 1.19 - 1.18 - 1.15 - 1.14 - 1.05 - 1.008.

According to the present author’s estimations, a coefficient of 1.15 would seem the most probable.

**Figure 8** , which is obtained from a method of calculation proposed by Silvester [19], makes it possible to obtain a satisfactory approximate value for any rectangular conductor.

As in the case of figure 6, the curves are established as a function of D.C. resistance Rc, and for 50 Hz.

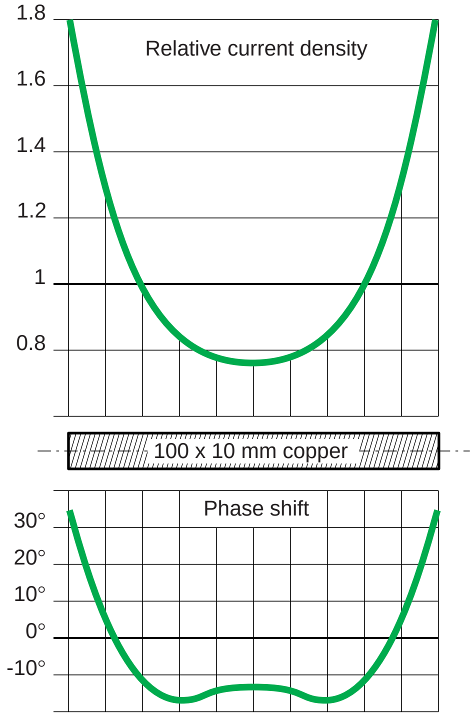


*Fig. 7 : current density and phase-shift in a rectangular bar.*

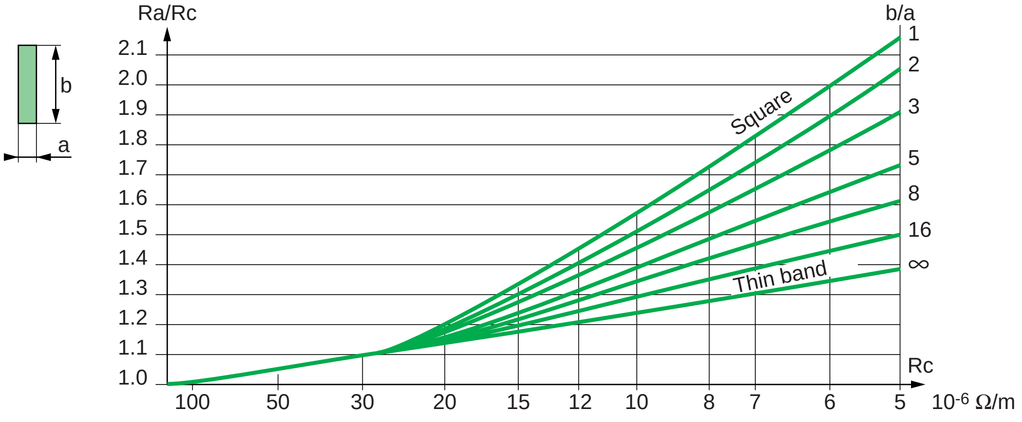


*Fig. 8 : extra loss coefficient for skin effect in rectangular conductors.*

# 3 Proximity effects

Up till now, the conductor subjected to the skin effect has been considered to be isolated and outside the influence of any magnetic field, except for its own. This assumption is no longer valid when another conductor is in the neighborhood; the field of each conductor perturbs the current distribution in the other by the so-called proximity effect.

This expression covers three similar phenomena which it appears necessary to dissociate in spite of their similarity for greater clarity:

- The direct proximity effect

This is the mutual influence of the respective current densities in the neighbouring conductors, carrying currents in the same direction.

- The inverse proximity effect

This is, on the contrary, the mutual influence of the respective current densities, in neighbouring conductors, carrying currents flowing in the opposite direction.

- The induced proximity effect

It characterizes the associated phenomena between the currents flowing in the conductor and the currents it induces in neighbouring metallic parts.

In reality, in a three-phase busbars with several bars per phase, these effects overlap as is shown in the heating experiment illustrated in **figure 9** ; the busbars in question has, per phase, 4 copper bars of 80 x 6 mm which are 6 mm apart; a distance of 60 mm separates the phases.

A point vertically above each of the 12 bars represents its heating above the ambient temperature, for a current of 2500 A.

Several aspects of the effects mentioned are found in this diagram, even though the heating does not represent exactly the density of the corresponding currents.

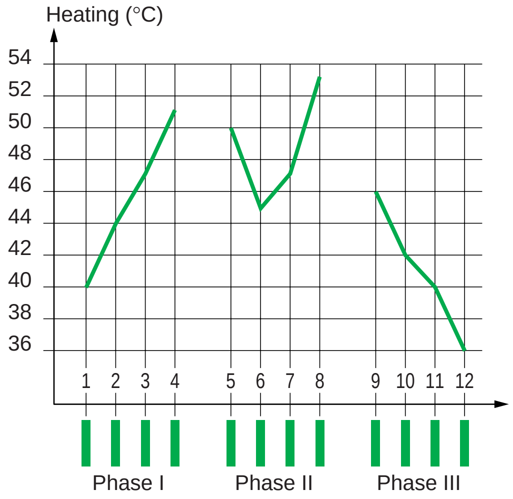


***Fig. 9*** *: heating of a 3-phase busbar.*

In particular it can be noted that:

- the 2 inside bars, 6 and 7, of the central phase heat up less than the exterior bars despite the latter better cooling,
- the asymmetric proximity effect resulting from the 120° phase difference for the three-phase current, manifests itself between bars 4 and 5 on the one hand 8 and 9 on the other hand.

This test shows also the ambiguity in the notion of the average heating up of a busbar on which values as different as 36° and 53° can be noted; these are much greater than the experimental effects.

Let us analyse now the three proximity effects separately, before quantifying in real cases the first two which are of most interest when they are combined with the skin effect.

## 3.1 The direct proximity effect

Consider a solid conductor of square cross-section (see **fig. 10** ). The current density is higher (a) on the periphery because of the skin effect.

Let the conductor be split longitudinally in two halves (b); no changes yet appear in the current distribution. If these two halves are gradually separated, (c), the magnetic field of each one changes and the current densities on the surfaces which face each other will increase until they become equal to those of the outside surfaces for a separation of about 2 to 3 times the length of the side of the square.

This effect becomes apparent, in addition to the skin effect, for bus bars of which each phase is made up of several flat bars which are electrically and spatially parallel.

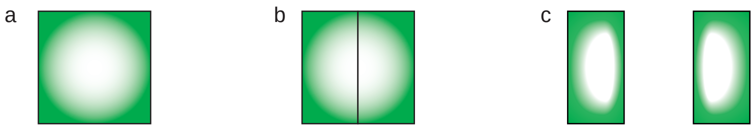


*Fig. 10*

**Figure 11** , which is taken from reference [5], refers to a group of four 100 x 10 mm copper bars, 10 mm apart. The upper curves give the relative current density at each point on the axis; the lower curves give the relative phase lead or lag with respect to the voltage at the extremities. In each case there are 2 curves, one for the outside bars, another for the inside bars.

It is fairly surprising to note the big difference in current density, in the ratio 8 to 1, as well as the phase difference of over 90° at the center of the internal bars.

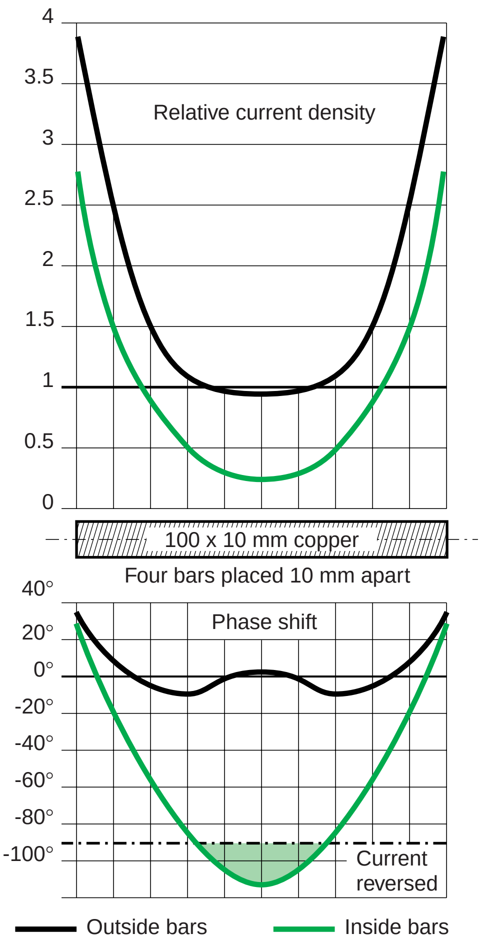


*Fig. 11 : current density and phase-shift in a group of 4 bars.*

**Figure 12** refers to a group of five 80 x 10 mm bars placed 10 mm apart [13]; the curves drawn on the cross section are lines of equi-current density.

It can be seen that in a batch of neighbouring bars the current distribution is only slightly different to that which would be produced merely by the skin effect in a solid conductor having the same external dimensions.

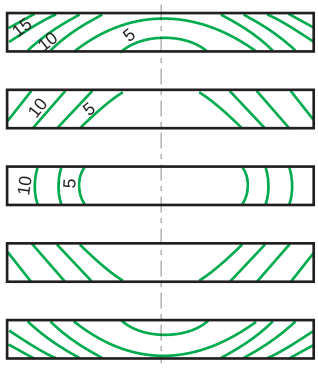


*Fig. 12 : current density in a group of 5 bars.*

The greater the number of parallel bars, the lower the amount of current carried by the inside, as opposed to the outside bars. This unequal distribution of current, leading to heating, is partially compensated (see fig. 9) by the improved cooling of the external bars.

For a sufficiently high separation of the bars, of the order of 3 times their largest dimension, the proximity effect disappears totally; for intermediate distances however, an appreciable decrease in the loss coefficient K occurs as can be seen from the tests and calculations [13] carried out on four 80 x 10 mm bars, placed 10 to 44 mm apart:

| **Distance in mm** | 10 | 20 | 40 |
|---|---|---|---|
| $K = \frac{Ra}{Rc}$ | 1.65 | 1.53 | 1.38 |

## 3.2 The reverse proximity effect

This effect, very similar to, but the reverse of the previous effect appears when two neighbouring conductors carry currents in opposite directions (see **fig. 13** ); an increase in the current density occurs on the inside surfaces which are the parts of the conductor for which the inductance is least.

In the connection of high current equipment, this effect is noted as soon as conductors of the same phase form a closed loop. In **figure 14** , the inside bars A of the loop carry a higher current than the outside bars C.

In 3-phase busbar system, the reverse proximity effect is quite marked for low voltages, when the bars are dose together. Whatever the order of the phases, there are always two neighbouring phases carrying opposing currents during a fraction of a period.

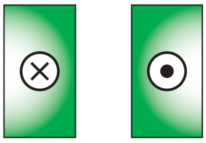


*Fig. 13*

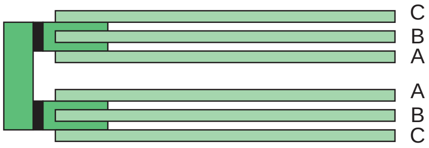


*Fig. 14*

## 3.3 The induced proximity effect

Metallic parts which are positioned close to conductors carrying a high alternating current are the seat of induced currents causing extra losses, whence an indirect increase in the effective resistance of the inducing circuit. Simultaneously, the latter’s inductance decreases.

When the part is made of steel, the internal losses are increased further by losses due to hysteresis. This is why it is not possible to use a protective steel casing for a single-phase conductor carrying more than about one hundred amperes, without risking that this casing is heated to over 100° C.

Some figures are given in references [15], [18] and [22] for typical cases of steel or aluminium parts placed close to conductors.

The induced currents which have been mentioned are not always parasitic. The field they produce is in opposition, by definition, to the main inducing field. Thus an aluminium plate which is close to a bus bars acts as a magnetic screen, reducing the field outside it to more or less zero.

This effect is particularly marked when each phase of a high current conductor (5000 to 30000 A) is enclosed in an aluminium sheath; if the sheaths are interconnected, a current flows which is almost equal to the main current. As a result the external field is suppressed, as are, in consequence, the electrodynamic stresses between the phases.
The design of the busbar with sheathing is not within the scope of this study; the reader can refer to numerous publications for more information on this subject [15], [16].

# 4 Effective resistance of busbars

## 4.1 Busbars made up of flat bars

The construction of busbar is usually carried out by putting together several flat bars in parallel for each phase. The spacing between the bars is made equal to their thickness for practical reasons, and this leads to skin and proximity effects as shown in figures 11 and 12.

If one refers to published results, no accurate quantitative estimations of these combined effects can be found. An order of magnitude forthe extra loss coefficient K is given in **figure 15**for 2 cross-sections of copper: 100 x 5 and 100 x 10 mm.

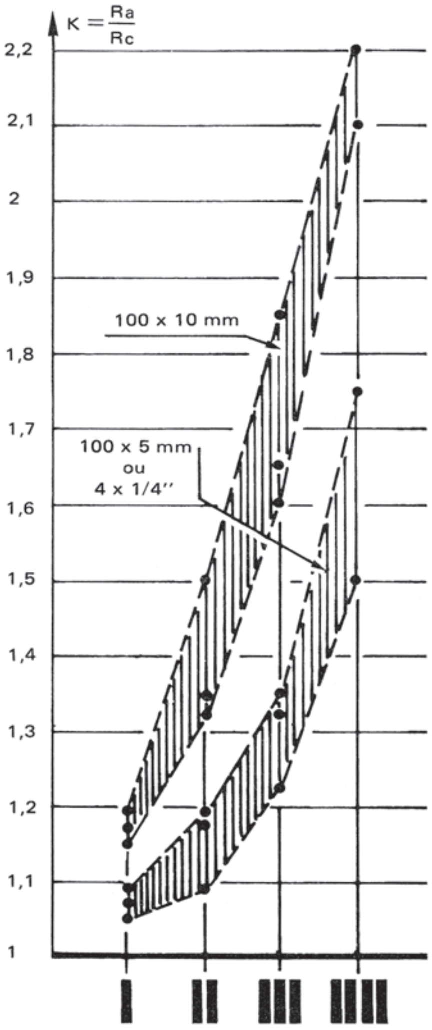


*Fig. 15 : extra loss coefficient in groups of flat bars.*

For each group of 1, 2, 3 or 4 bars, points corresponding to published results encircle a shaded area in which the probable value of K must be situated.

In the absence of any more accurate results, the search for a value of K for a set of bars of any size can be made using the curves of figure 8, and equating the set to a single bar of the same height but of a width equal to the overall width of the whole set. The resistance Rc is the equivalent to that of all the bars in parallel.

The coefficient K is here found by excess, but this extrapolation is only valid for bars which are not separated by more than their thickness.

In effect, a generous spacing and a judicious positioning of the bars lead to a reduction in the loss coefficient; for example in **figure 16** [22] are shown the coefficients K for groups of 3, 4, 6 and 8 bars of 100 x 6 mm; the closest bars are 6 mm apart, the furthest 60 mm.

The relative gain on the losses is 20% for 3 bars and 40% for 4 bars, according as to whether they are in one or two batches.

It is rare that the use of 5 bars grouped together is considered because of the high loss coefficient caused by inadequate use of the central bar.

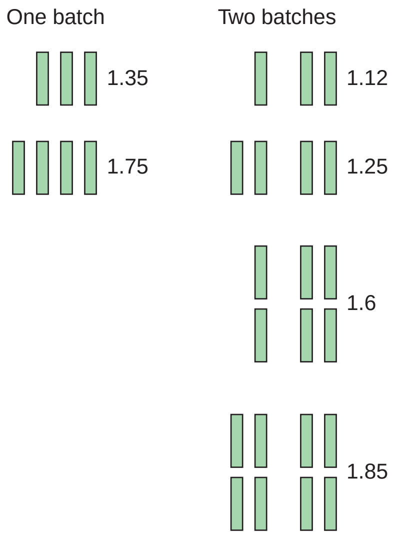


*Fig. 16 : extra loss coefficient in groups of 3 to 8 flat bars according to their disposition.*

It has also been proposed to place the 4 bars of a phase along the edges of a square, a solution which gives the advantages of a tubular conductor, but the supports and the tappings become fairly complicated here.

All these indications concern the skin effect acting simultaneously with the proximity effect, in a group of several bars of the same phase; for 3 phases, if the distance between the closest bars of 2 different phases is less than twice the height of these bars, an inverse proximity effect acts in addition to the two previous effects.

In order to obtain a value for the coefficient K giving the increase in the corresponding losses, one should refer to the DIN standards no. 43.671 [23] which give the coefficient K4 for bars of 5 or 10 mm thickness, or to reference [24] in which the average geometric distances of various shapes of conductor make it possible to carry out the relevant calculation.

An arrangement which is of particular interest in the case of 3 phase system is the so-called sandwich: busbars inter-twined or permuted [11]. The bars of each of the phases are not placed in independent groups for each phase, but are on the contrary placed in between each other.

A busbar having 2 bars per phase (J, R, V) is thus arranged as in **figure 17** ; the proximity effects are eliminated, the current density in each bar is almost identical and the coefficient K is little over 1.

Two disadvantages limit the general use of this process: certain complications in the connection and joints, and the difficulty of obtaining isolation of the phases, even at low voltages.

An additional advantage is the reduction in the electrodynamic stresses, to which can be added a decrease in the inductance per phase by a factor of 10 ; this last characteristic of sandwich type busbars has a favorable effect on the induced voltage drop in normal operation, but leads to an increase in the value of short-circuit current.

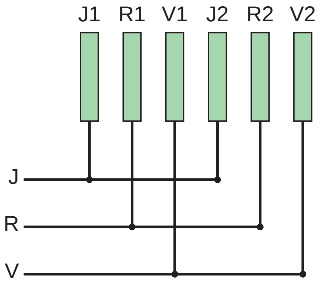


***Fig. 17***

## 4.2 Minimal heating, or reduction in the extra losses?

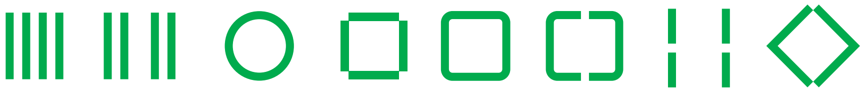
Up till now the effects mentioned have only been analysed from the point of view of the increase in the effective A.C. resistance, that is of the extra losses produced by the Joule effect.

The normal consequence is increased heating of the conductors, but this is sometimes compensated for by adopting an arrangement which favors cooling by radiation or convection.

Now the heating is at present time the only important criterion taken into consideration for the designing of a high current conductor; however the minimal heating is not always found in conjunction with the lowest loss coefficient; it can be seen from figure 15 that the coefficient K is more or less the same for one 100 x 10 bar or two 100 x 5 bars, but because of the larger cooling surface in the latter case, it is possible, at equivalent heatings, to obtain a current of 10% greater, and thus losses which are 20% greater.

Another characteristic example is the tubular conductor whose optimized shape guarantees a coefficient K close to 1; however this tube has the smallest cooling surface (without any forced ventilation on the inside) and it can be seen in **figure 18** that it is far from having the profile which carries the highest current, for a heating and a cross-section the same as other configurations.

The designer of a high current conductor will sometimes be advised to choose a technology not only according to the heating produced, but also according to the total losses.



| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| I | 1 | 1.18 | 1.25 | 1.28 | 1.50 | 1.54 | 1.57 | 1.71 |
| K | 1.75 | 1.25 | 1.05 | 1.1 | 1.08 | 1.15 | 1.3 | 1.1 |
| P | 1.75 | 1.75 | 1.65 | 1.8 | 2.4 | 2.7 | 3.2 | 3.2 |

***Fig. 18*** *: comparison of profiles having equal total cross-section.*

## 4.3 Busbars having special profiles

When the current to be carried exceeds 4 to 5000 A, the busbar made up of flat bars become insufficiently adapted unless spacings are adopted as shown in figure 16. In this case special profiles are used which satisfy better the following two criteria simultaneously:

- efficient cooling
- reduced losses.

It is also necessary that these profiles have a good resistance to electrodynamic stresses on short-circuiting, and also be easy to install.

- The much used U profile, set up in pairs, replies satisfactorily to these criteria on the whole (see **fig. 19** ).
- Profiles consisting of angle irons in pairs are more efficient electrically and thermally, but are a bit less practical.
- Another system which is used, in particular in the USA where it is standardized [21], is a square profile with rounded corners whose loss coefficient is nearly equal to that of the tube and whose cooling is much improved by holes which are placed in a staggered form on two horizontal sides.
- A European manufacturer uses also a V-shaped 120° angle profile assembled in groups of 2, 3 or 4 per phase [17].

The choice amongst these different profiles is a compromise between:

- the heating (for equal cross-sections),
- the electrodynamic strength,
- the over-all dimensions,
- the ease of connection,
- the simplicity of the insulating supports,
- the losses,
- the cost of the metal used.

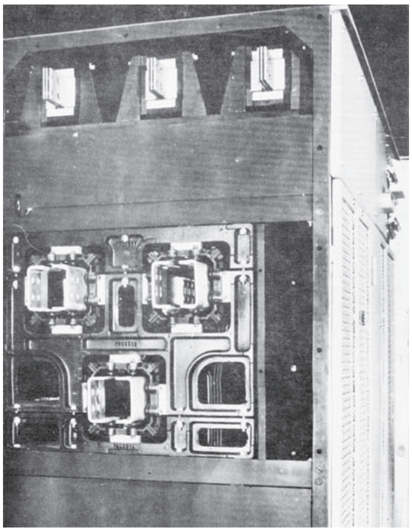


***Fig. 19** : Liner Norway - 5000 A 3-phase busbar 250 kV peak electrodynamic strength.*

A classification based on the most important criteria -heating-, is given in figure 18 (see [25]).

All these profiles or assemblies have the same total cross-section: 4 square inches or 2850 $mm^2$ of copper. For equal heating, figure 1 indicates for each one the relative currents which can be carried with respect to the most unfavourable case of 4 closely placed bars.

The classification becomes different if it is based on the extra loss coefficient K or on the total loss factor P obtained by multiplying K by $I^2$.
The conclusions are easily derived from a comparison of these three criteria.

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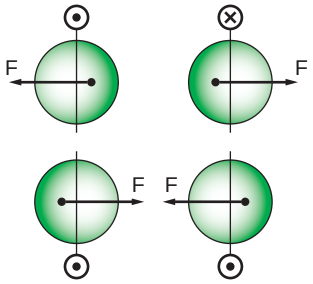
**A paradox of electromagnetism?**



One deduces from the laws of electromagnetism that two neighbouring conductors carrying currents flowing in the same direction will attract each other. On the other hand, the direct proximity effect implies that a higher current density is produced in those parts of the conductors which are furthest apart, as if the elementary current filament repulsed each other.

The same contradiction is apparent between the inverse proximity effect, which apparently results in an attraction between the currents, and the repulsion force between opposing currents. The paradox is only apparent since the nature of the two phenomena are very different:

The proximity effect causes an excess current density in the region of minimal induction, containing the least flux; on the other hand, the electrodynamic stresses act in such a way as to increase the electromagnetic energy accumulated in the circuit by the inductance, which increases in the same proportions.

Other characteristics differentiate these two phenomena:

The proximity effect is independent of the current value, and only occurs in a variable regime; it depends on the resistivity and on the frequency. A consequence of this analogy should however be noted:

The point of application of attractive or repulsive forces is not the geometrical center of conductors subjected to the proximity effect, but it moves towards regions of maximum current density ; it is necessary to take this into account in calculating the electrodynamic stresses between closely neighbouring conductors.

## 4.4 Resistivity of the metal, copper or aluminium?

It was assumed in what has preceded that the metal used was copper; it now should be noted that the skin and proximity effects become more marked as the resistance decreases.

A copper conductor will thus have a higher loss coefficient than the same conductor made of aluminium, but the latter, requiring a cross-section 1.6 times greater to obtain the same resistance (for a direct current), losses this advantage over the copper conductor because the two conductors have the same coefficient K (see fig. 6 and 8) for the same shape and the same resistance Rc.

In practice, the replacement of copper by aluminium is not done on the basis of equivalent resistance or voltage drop but rather on that of equivalent heating; this amounts to multiplying the cross-section by 1.4 to 1.5 only, in order to take into account the improved cooling of the larger surface.

To sum up, for equal heating, an aluminium conductor has a better loss coefficient than an equivalent copper conductor; it must not be forgotten however that this entails higher losses which must be evacuated and also paid for. The price per kilo and the much lower density of aluminium are the determining factors which lead to the metal being chosen to high current intensity conductors.

## 4.5 Influence of the frequency

Only the industrial frequency of 50 Hz has been taken into consideration in the preceding calculations; their accuracy which is only relative however makes them valid for frequencies of up to 60 Hz. Figures 6 and 8 which give the skin effect coefficients for tubes and flat rods at 50 or 60 Hz can be used for any other frequency with the corrections given. Amongst these frequencies 25 Hz is hardly ever used; as for 16 2/3 Hz it can be assimilated to direct current.

Serious problems of skin effect are set by the 400 Hz frequency which is used for special circuits (marine, aviation, etc.), as soon as the current reaches a few hundred amperes: the “skin” of the copper is reduced to 3 mm at this frequency.

In industrial networks, harmonic currents having frequencies which are multiples of 50 Hz (the harmonics 3 and 11 cause the most nuisance) can be superposed to the fundamental frequency. These currents encounter an effectively increased resistance and significant losses and heating occur.

# 5 Skin and proximity effects in the transient state

These effects, which are the result of rapid fluctuations of the current and therefore of the flux in a conductor, appear both during a periodic variation (the case of A.C. in a stable regime) and during a transient change (the case of the sudden appearance of a high short-circuit current).

Without going into too much detail, it should be remembered that the skin effect has an unfavourable influence on the establishment of a direct current.

The rate of rise expressed as $\frac{di}{dt}$ is inversely proportional to the time constant $\frac{L}{R}$ of the circuit. The skin effect causes a decrease in L and an increase in R, and thus the transient time constant is lower, and the short-circuit current growth faster.

These factors must be taken into consideration in high current D.C. installations (electrochemical) which use solid conductors of large cross-section and for which the transient skin effect can be very large.

In A.C. networks, the establishment of a short-circuit results in an asymmetric regime on certain phases, due to a d.c. component whose dampening, proportional to $\frac{L}{R}$, is faster in this case.

**Discovery of the transient skin effect**

The anecdote told by Arago occured round about 1880. A worker who is isolated with respect to the ground has a big iron bar in his hands. He is about to use it to cause a short circuit between two terminals of a high capacity gramme dynamo used in electroplating.

As he establishes contact, he suddenly drops the bar which he claims has burnt his hands. The bar is immediately picked up very carefully, but is surprisingly almost cold.

What in fact actually happened?
We now know that the skin effect in steel causes a rapidly changing current to be localised in a very thin peripheral layer which, alone, heats up instantly, really burning the hands of the worker.

Less than a minute later, the heat had diffused into the bulk of the bar which was no more than luke-warm.

# 6 Conclusion

The skin and proximity effects in high current density conductors are complex phenomena whose repercussions are often under-estimated or neglected because of the difficulty of quantifying them accurately.

This article has intentionally given more importance to practical data and results with respect to the theoretical aspects of the phenomena, it has thus led to two important points being made apparent, indirectly:

- The lack of accuracy in the data published on the effective resistance of some type of busbars frequently employed. A higher accuracy should be obtained if persons having carried out calculations or experiments could look again at some old work, most of it dating back several decades, and apply powerful modern methods such as the use of computers and of electronic measuring equipment.
- The taking into account of the cost of the total losses (normal losses and extra losses in A.C.) to evaluate the economics of the bus bars to be used. This techno-economic approach can lead those exploiting networks operating almost continually close to their rated capacity, to invest more, initially, in a bus bars which is better designed, in order to waste less energy over the several years of operation.

### Author's note

It is appropriate to pay tribute to the scientists and engineers who discovered and created the laws which govern the skin effect: Lord Kelvin then P. Boucherot and especially H.B. Dwight (USA) as well as H. Schwenkhagen (Germany) who, at the start of the 20th, carried out the necessary long and difficult calculations, without the help of present day electronic computers.

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**Schneider Electric**

Direction Scientifique et Technique,
Service Communication Technique
F-38050 Grenoble cedex 9
Télécopie : 33 (0)4 76 57 98 60
E-mail : fr-tech-com@mail.schneider.fr

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